## ❖ CHOOSING INTEGERS

- How to choose three consecutive integers?

  $x, \quad x+1, \quad x+2$

- How to choose three consecutive even integers: $x,\ x+2,\ x+4$
- How to choose three consecutive odd integers: $x,\ x+2,\ x+4$

  (It is the same as even integers!)

## ❖ SEQUENCES & SERIES

- $n$th term of an Arithmetic Sequence $\{a_n\}$ with first term $= a_1$ and common difference $= d$ is

  $$a_n = a_1 + (n-1)d$$

- $n$th term of a Geometric Sequence $\{a_n\}$ with first term $= a_1$ and common ratio $= r$ is

  $$a_n = a_1 \cdot r^{n-1}$$

- Sum of the first $'n'$ terms of an Arithmetic Sequence $\{a_n\}$ with first term $= a_1$ and common difference $= d$ is

  $$S_n = \frac{n}{2}(a_1 + a_n)$$

  Or

  $$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

- Sum of the first $'n'$ terms of a Geometric Sequence $\{a_n\}$ with first term $= a_1$ and common ratio $= r$ is

  $$S_n = \frac{a_1(1-r^n)}{1-r}$$

- Sum of an Infinite Geometric Series

  $$S_n = \frac{a_1}{1-r} \quad \text{if } |r| < 1$$

## ❖ PERMUTATIONS & COMBINATIONS

- *'n'* factorial refers to the product of the first *'n'* natural numbers

$= n!$

$= n(n-1)\ldots 3 \cdot 2 \cdot 1$

- *nPr* refers to the number of different arrangements of *'r'* things taken at a time out of the given *'n'* things. This is when order of the things is matters and is important.

  $$nPr = \frac{n!}{(n-r)!};\ (r \le n)$$

- *nCr* refers to the number of different selections of *'r'* things taken at a time out of the given *'n'* things. This is when order of the things does not matter and is not important.

  $$nCr = \frac{n!}{r!\,(n-r)!}\ ;\ (r \le n)$$

## ❖ PROBABILITY

- Probability of an Event $A$ is defined as the ratio of the number of favorable outcomes to Event $A$ to the number of possible outcomes and is denoted as $P(A)$

$$P(A) = \frac{\text{number of favorable outcomes to Event } A}{\text{number of possible outcomes}}$$
$$= \frac{n(A)}{n(S)}$$

- Probability of the Event complementary to Event $A$ is denoted as Event $\overline{A}$ is defined as the ratio of the number of unfavorable outcomes to Event $A$ to the number of possible outcomes and is denoted as $P(\overline{A})$

$$P(\overline{A}) = \frac{\text{number of unfavorable outcomes to Event } A}{\text{number of possible outcomes}}$$
$$= \frac{n(\overline{A})}{n(S)}$$

- The sum of the probabilities of the original event $A$ and the complementary event $\overline{A}$ is always equal to one.

  $P(A) + P(\overline{A}) = 1$ or

  $P(\overline{A}) = 1 - P(A)$ or

  $P(A) = 1 - P(\overline{A})$

- **Addition Theorem of Probability:** If $A$ and $B$ are any two events then the probability of the happening of *at least one* of the events is defined as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- **Multiplication Theorem of Probability:** If $A$ and $B$ are any two independent events then the probability of the simultaneous happening of *both* the events is defined as

$$P(A \cap B) = P(A) \cdot P(B)$$

## ❖ STATISTICS

- Mean *or* Arithemtic Mean *or* Average $= \dfrac{\text{Sum of the } n \text{ values}}{n}$
- Median refers to the value of the middlemost item in the distribution when all items are arranged either in the ascending order or descending order.
- Mode refers to the value of the most repeated item in the distribution.
  Range is the difference between the greatest and the smallest data values.
- Standard Deviation is a value that measures how far apart the individual data scores are from the mean. More the spread in the distribution of the values around the mean, higher is the standard deviation and vice-versa.
- A sample is a representative part of the total population. Margin of Error for a survey based on the sample is a value that

estimates the extent to which the conclusions drawn may differ from the results that would have been obtained from surveying the entire statistical population.

- A Confidence Interval for a sample statistic such as the mean is the range of values for which we expect the true population to lie within.
- Statistical Bias: This happens when you are selecting your sample or your data wrong. Usually, this means accidentally working with a specific subset of your audience instead of the whole, rendering your sample unrepresentative of the whole population.
- Weighted Average (Arithmetic Mean) Formula: If one group has $N_1$ values with an average $\overline{x}_1$ and another group has $N_2$ values with an average $\overline{x}_2$, then the combined arithmetic mean of the pooled or combined grouped is $\overline{x}_{12}$ and is found by using the formula –

$$\overline{x}_{12} = \frac{N_1 \cdot \overline{x}_1 + N_2 \cdot \overline{x}_2}{N_1 + N_2}$$

## ❖ PROPORTIONS

- Cross-Product Property of Proportions

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{then } a \cdot d = b \cdot c$$

- Reciprocal Property of Proportions

$$If\ \frac{a}{b} = \frac{c}{d}\ then\ \frac{b}{a} = \frac{d}{c}$$

## ❖ PERCENTAGE

- The statement '*a* is *p*% of *b*' can be converted into a proportion

$$\frac{p}{100} = \frac{a}{b}$$

If any two values out of the three values $'a'$, $'p'$ or $'b'$ are given, then we can find the remaining third missing value.

- Finding Original Amount After a Percent Increase

$$\text{Original Amount} = \frac{New\ Amount\ after\ an\ increase\ of\ p\%}{(100\ +\ p)\%}$$

- Finding Original Amount After a Percent Decrease

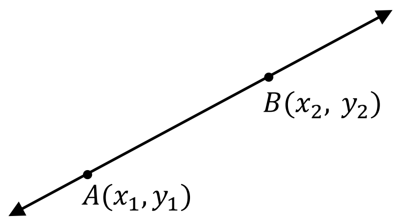
$$\text{Original Amount} = \frac{New\ Amount\ after\ a\ decrease\ of\ p\%}{(100 -\ p)\%}$$

- Finding the Percent of Increase or Decrease

$$\text{Percent of Change } = \frac{Amount\ of\ Change}{Original\ Amount} \text{ X } 100$$

## LINEAR FUNCTION

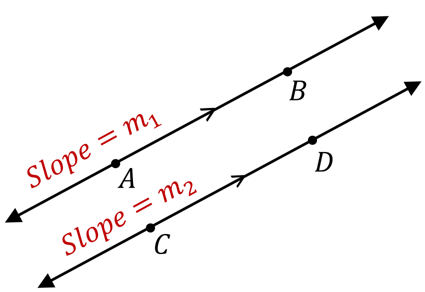
- *Slope of a line*




- Slope of a line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$

$$= m = \frac{Rise}{Run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

- *Slope of Parallel Lines*

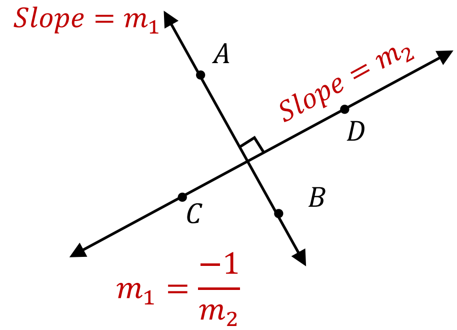



- Parallel lines have equal or same slope

Slope of line $\overleftrightarrow{AB}$ = Slope of line $\overleftrightarrow{CD}$

$$m_1 = m_2$$

- *Slope of Perpendicular Lines*




Perpendicular lines have negative reciprocal slopes

$$m_1 = \frac{-1}{m_2}$$

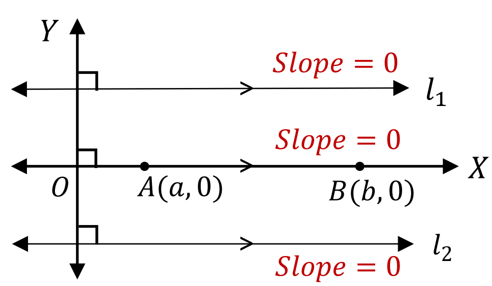
Or

Product of the slopes is $-1$

Or

$$m_1 \cdot m_2 = -1$$

- *Slope of the $X - axis$* and Horizontal Lines

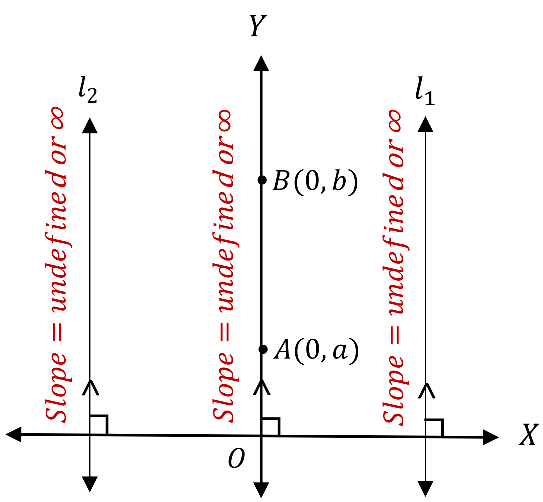



- Slope of the $X - axis$ or a horizontal line or a line parallel to the $X - axis$ is zero or has no slope
- Slope of the $X - axis$ *or* line $\overleftrightarrow{AB}$

$$= \frac{0-0}{a-b} = \frac{0}{a-b} = 0$$

- Lines $l_1$ and $l_2$ are horizontal and parallel to the $X - axis$
- Slope of line $l_1$ = Slope of line $l_2 = 0$

- *Slope of the $Y - axis$ and Vertical Lines*

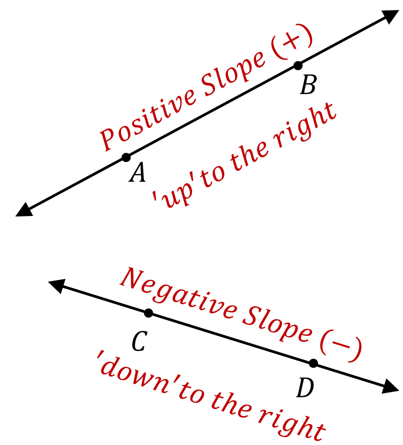



- Slope of the $Y - axis$ or a vertical line or a line parallel to the $Y - axis$ is undefined or infinity ($\infty$)
- Slope of the $X - axis$ $or$ line $\overleftrightarrow{AB}$

$$= \frac{a-b}{0-0} = \frac{a-b}{0} = \text{undefined } or\ \infty$$

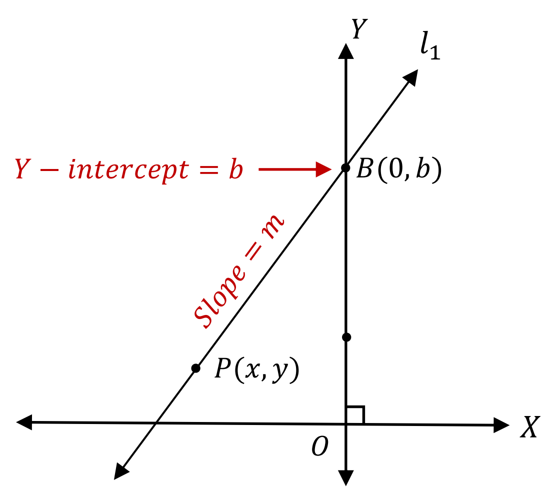
- Lines $l_1$ and $l_2$ are vertical and parallel to the $Y - axis$
- Slope of line $l_1$ = Slope of line $l_2$ = undefined $or$ $\infty$ (infinity)

- *Positive Slope & Negative Slope*




- Slope of a line sloping up to the right is positive. Slope of line $\overleftrightarrow{AB}$ is positive $(+)$
- Slope of a line sloping down to the right is negative. Slope of line $\overleftrightarrow{CD}$ is negative $(-)$

- *Slope-Intercept Form of a Line*

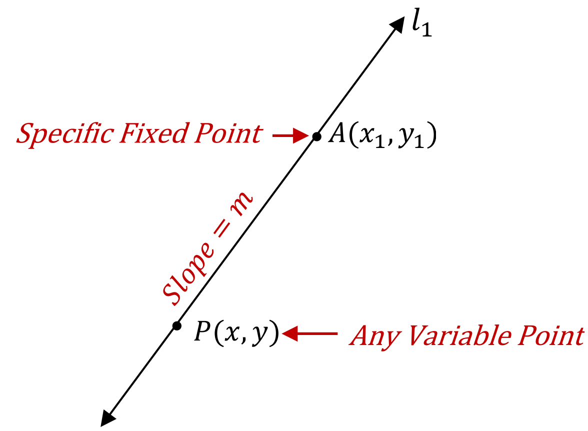



- $P(x, y)$ is any variable (moving) point on the line $l_1$

- Equation of line $l_1$ with slope $= m$ and $y$-intercept $= b$ in SLOPE-INTERCEPT FORM is
  $y = mx + b$

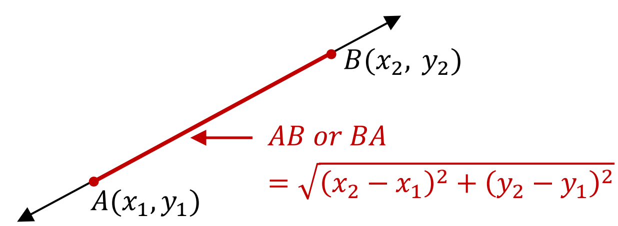
- *Interpreting $y = mx + b$*
- The constant $'m'$ indicates for each unit change in $'x'$ how many units change in $'y'$ results.
- In a linear function, the value of $'y'$ changes by a constant amount for every *(one)* unit change in of $'x'$. That constant amount is $'m'$.
- If $m > 0\ (+)$, the value of $'y'$ increases by $'m'$ units for every *(one)* unit increase in $'x'$.
- If $m < 0\ (-)$, the value of $'y'$ decreases by $'m'$ units for every *(one)* unit increase in $'x'$.
- The constant $'b'$ represents some starting value of $'y'$ or some initial condition. It is the value of $'y'$ when $x = 0$ .

- *Point-Slope Form of a Line*



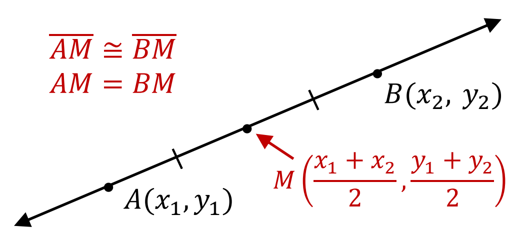

- Equation of the line passing through a point $A(x_1, y_1)$ and having slope $= m$ in POINT – SLOPE FORM is
  $$y - y_1 = m(x - x_1)$$

- *Distance Formula*




- Distance from $A(x_1, y_1)$ to $B(x_2, y_2)$
  $= AB\ or\ BA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

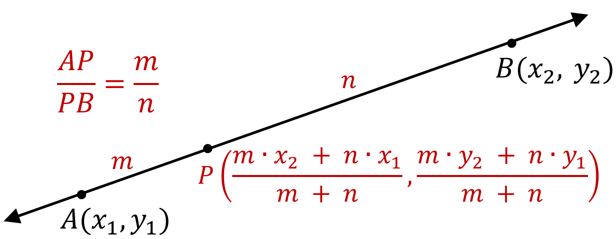
- *Midpoint Formula*



Midpoint between $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- *Section Formula (Internal Division)*

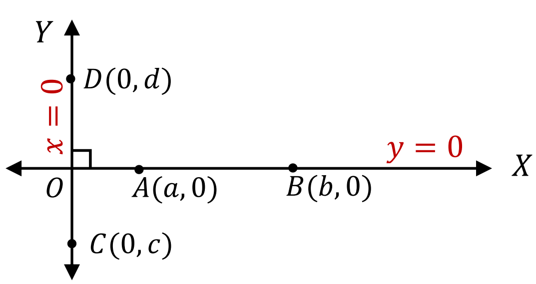



If $A(x_1, y_1)$ and $B(x_2, y_2)$ are the end points of $\overline{AB}$ and $P(x, y)$ divides $\overline{AB}$ internally in the ratio of $m:n$ then the coordinates of the Point of Division $P$ can be found by using the formula based on weighted average:

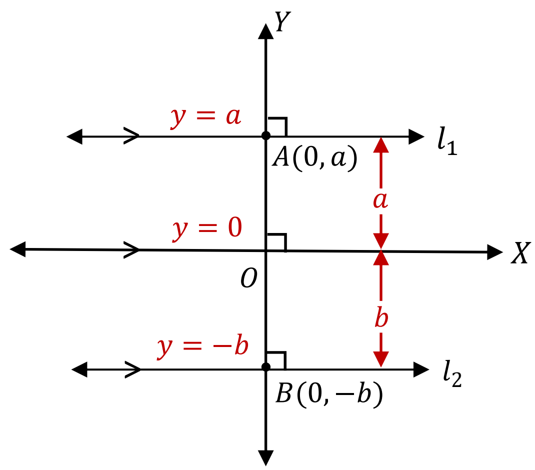
$$x = \frac{m \cdot x_2 + n \cdot x_1}{m + n}$$

$$y = \frac{m \cdot y_2 + n \cdot y_1}{m + n}$$

- *Equation of the Coordinate Axes*

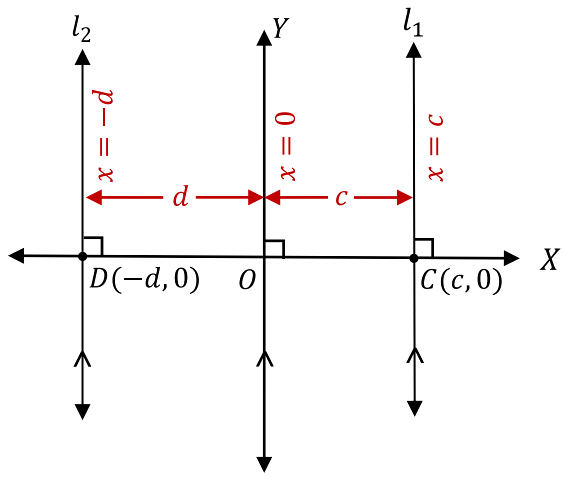



- Equation of the $X - axis$ is $y = 0$. This is because every point on the $X - axis$ has its $'y'$coordinate $= 0$. The $'x'$coordinate is any real number.
- Equation of the $Y - axis$ is $x = 0$. This is because every point on the $Y - axis$ has its $'x"$ coordinate $= 0$. The $'y'$coordinate is any real number.
- *Equation of lines parallel to the $X - axis$*




- Equation of horizontal line $l_1$ passing through $A(0, a)$ is $y = a$. In other words, equation of a line parallel to the $X - axis$ and $'a'$ units *above* the $X - axis$ is $y = a$.
- Equation of horizontal line $l_2$ passing through $B(0, -b)$ is $y = b$. In other words, equation of a line parallel to the $X - axis$ and $'b'$ units *below* the $X - axis$ is $y = b$.

- *Equation of lines parallel to the $Y-axis$*




- Equation of vertical line $l_1$ passing through $C(0,c)$ is $x=c$. In other words, equation of a line parallel to the $Y-axis$ and $'c'$ units to the *right* of the $Y-axis$ is $x=c$.
- Equation of vertical line $l_2$ passing through $D(0,d)$ is $x=-d$. In other words, equation of a line parallel to the $Y-axis$ and $'d'$ units to the *left* of the $Y-axis$ is $x=-d$.

- *Comparison between Linear and Exponential functions*

- A Linear function changes at a constant fixed amount per unit interval.
- An Exponential function changes by a common ratio over equal intervals.

- *System of Linear Equations – Number of Solutions*

If the equations of two lines are

$$a_1x+b_1y=c_1$$
$$a_2x+b_2y=c_2$$

There are three possibilities –

1. Intersecting Lines
2. Parallel Lines
3. Overlapping Lines

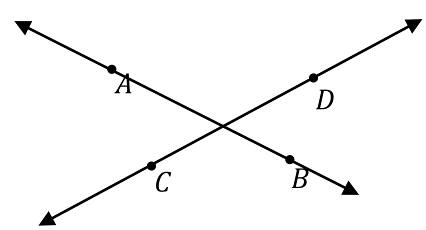
- *Intersecting Lines*

These lines have one point of intersection if the Ratio of the Coefficients is unequal

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\overset{?}{=}\frac{c_1}{c_2}$$

(different slopes $m_1\neq m_2$ and maybe different $y-$intercepts)

*Intersecting Lines* $(m_1\neq m_2)$



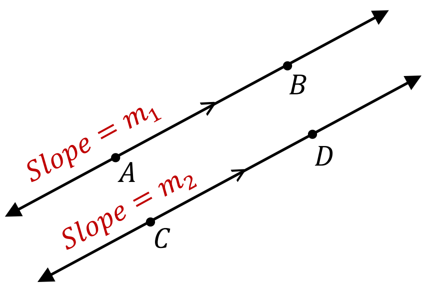

- *Parallel Lines*

These lines are parallel and have no point of intersection if Ratio of the Coefficients is equal but not equal to the Ratio of Constants

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(same slope but different $y$-intercepts)

*Parallel Lines* $(m_1 = m_2)$



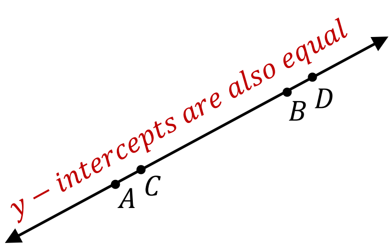

- *Overlapping Lines*

  These lines are overlapping and have infinite points of intersection if Ratio of the Coefficients is equal to the Ratio of Constants

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(same slope and same $y$-intercept)

*Overlapping Lines* $(m_1 = m_2)$




- $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ overlap and look like a single line.

❖ **RULES OF EXPONENTS**

- PRODUCT RULE

  $a^x \cdot a^y = a^{x+y}$

- QUOTIENT RULE (if $x > y$)

  $\frac{a^x}{a^y} = a^{x-y}$

- QUOTIENT RULE (if $y > x$)

  $\frac{a^x}{a^y} = \frac{1}{a^{y-x}}$

- NEGATIVE EXPONENT

  $a^{-x} = \frac{1}{a^x}$

- ZERO EXPONENT

  $a^0 = 1$

- POWER OF A POWER RULE

  $(a^x)^y = a^{xy}$

- DISTRIBUTIVE PROPERTY FOR EXPONENT OF PRODUCT

  $(ab)^x = a^x \cdot b^x$

- DISTRIBUTIVE PROPERTY FOR EXPONENT OF QUOTIENT

  $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

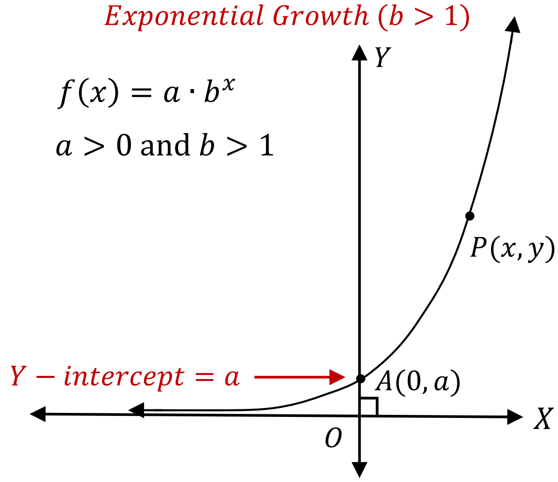
- If $x^a = x^b$, since the bases are the same, by comparing the exponents on the two sides $a = b$.

- Scientific Notation: A number is written in scientific notation if it is of the form $c \times 10^n$, where $1 \leq c < 10$ and $'n'$ is an integer.

## ❖ EXPONENTIAL FUNCTION

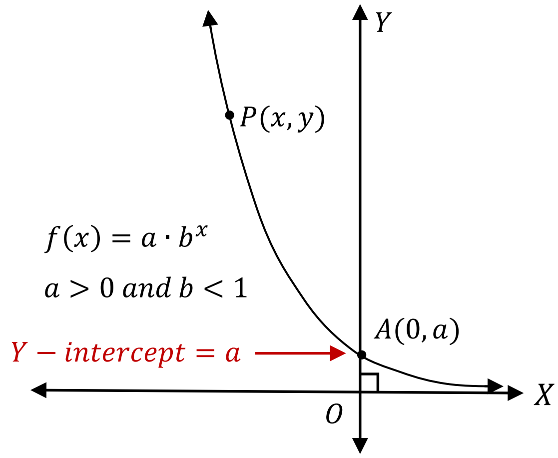
- **Exponential Function:** A function that involves the expression $b^x$ where $b > 0$ but $b \neq 1$
- **Exponential Growth Function:** A function of the form $f(x) = a \cdot b^x$ where $b > 1$
- **Growth Factor:** The quantity $1 + r$ in the exponential growth model $y = a(1+r)^t$ where $'a'$ is the initial amount and '$r$' is the percent increase expressed as a decimal.

*Exponential Growth* $(b > 1)$




- **Exponential Decay Function:** A function of the form $f(x) = a \cdot b^x$ where $a > 0$ and $0 < b < 1$
- **Decay Factor:** The quantity $1 - r$ in the exponential decay model $y = a(1-r)^t$ where $'a'$ is the initial amount and $'r'$ is the percent decrease expressed as a decimal.

*Exponential Decay* $(b < 1)$




## ❖ RATIONAL EXPONENTS

- CONVERTING A RADICAL EXPRESSION INTO RATIONAL EXPONENT (IN GENERAL)

  $\sqrt[m]{a^n} = a^{\frac{n}{m}}$
- CONVERTING A RADICAL SQUARE ROOT INTO RATIONAL EXPONENT

  $\sqrt{a} = a^{\frac{1}{2}}$
- CONVERTING A RADICAL CUBE ROOT INTO RATIONAL EXPONENT

  $\sqrt[3]{a} = a^{\frac{1}{3}}$
- DISTRIBUTIVE PROPERTY FOR PRODUCT

  $\sqrt[m]{ab} = \sqrt[m]{a} \cdot \sqrt[m]{b}$
- DISTRIBUTIVE PROPERTY FOR PRODUCT (SQUARE ROOT)

  $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

- DISTRIBUTIVE PROPERTY FOR QUOTIENT

$$\sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$$

- DISTRIBUTIVE PROPERTY FOR QUOTIENT (SQUARE ROOT)

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

## ❖ FACTORING & FOIL

- DISTRIBUTIVE PROPERTY

$$a(b+c) = ab + ac$$

- FOIL

$$(a+b)(c+d)$$
$$= a(c+d) + b(c+d)$$
$$= ac + ad + bc + bd$$

- PERFECT SQUARE OF A BINOMIAL SUM

$$(a+b)^2 = a^2 + 2ab + b^2$$

- PERFECT SQUARE OF A BINOMIAL DIFFERENCE

$$(a-b)^2 = a^2 - 2ab + b^2$$

- DIFFERENCE OF TWO SQUARES

$$a^2 - b^2 = (a+b)(a-b)$$

- CUBE OF A BINOMIAL SUM

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

- CUBE OF A BINOMIAL DIFFERENCE

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

- SUM OF TWO CUBES

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

- DIFFERENCE OF TWO CUBES

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

- $a^2 + b^2 = ?$

*(There is NO Formula for SUM OF TWO SQUARES)*

## ❖ QUADRATIC FUNCTION

- **Quadratic function:** There are three ways of expressing a quadratic function
  1. Standard Form
  2. Vertex Form
  3. Intercept Form

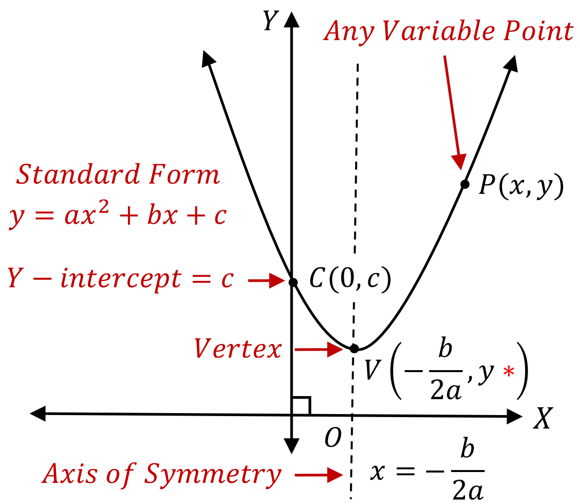
- *Standard Form of a Quadratic Function*
- A function of the form

$$y = ax^2 + bx + c$$

where $a \neq 0$.

The $y$-intercept of the function is $'c'$.

The graph of the function is either U – shaped ($a > 0$), or inverted U – shaped (∩) ($a < 0$)




- *To find the $y -$ coordinate of the vertex, first find the $x -$ coordinate by using the formula

$$x = -\frac{b}{2a}$$

and then plug the $x -$ value found above in the equation $y = ax^2 + bx + c$

- If the constant $a > 0$, the parabola opens upwards (U) and the turning point or the vertex of the parabola is the lowest point and its $y$-coordinate is the minimum value of the quadratic function.
- If the constant $a < 0$, the parabola opens downwards (∩) and the turning point or the vertex of the parabola is the highest point and its $y$-coordinate is the maximum value of the quadratic function.
- The $x$ coordinate of the vertex is

$$x = \frac{-b}{2a}$$

- The axis of symmetry of the parabola that passes through the vertex is a vertical line

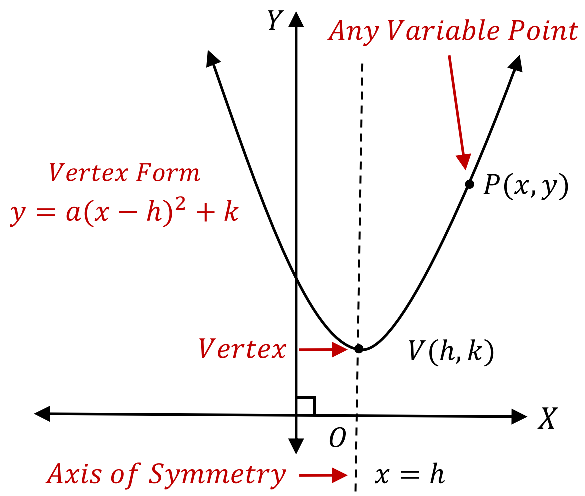
$$x = \frac{-b}{2a}$$

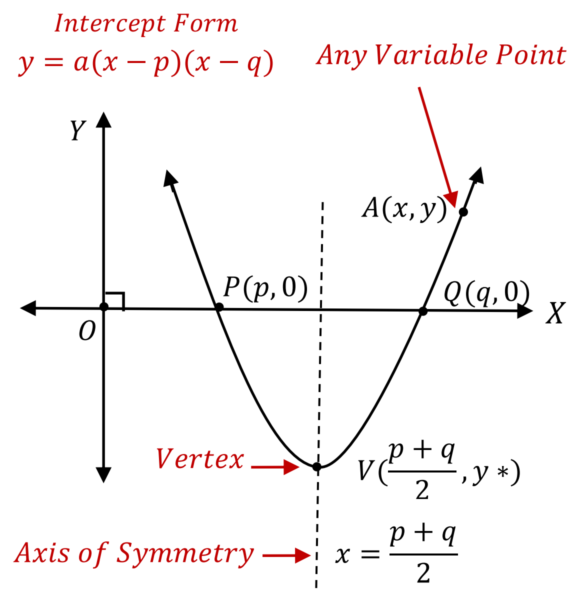
- *Quadratic Formula*
- If $ax^2 + bx + c = 0,$ where $a \neq 0$ and $a, b$ and $c$ are real numbers, then the roots or the solutions of the equation refer to the values of $x$ that satisfy the equation and are found by using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

TIP: The Quadratic Formula is especially useful to find the solutions where the quadratic equation is not factorable (the quadratic expression is prime) or where the values of $a, b$ & $c$ are not numeric.

- *Vertex Form of a Quadratic Function*
- Equation of the parabola in the Vertex Form is $y = a(x - h)^2 + k$ where the vertex is $(h, k)$ and the equation of the axis of symmetry is $x = h$

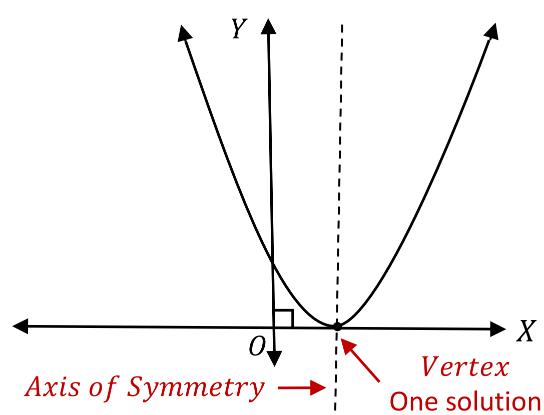


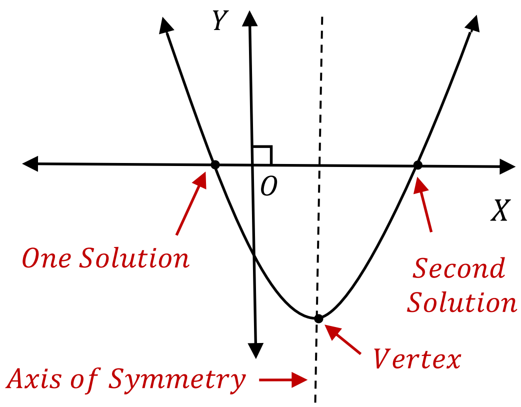

- *Intercept Form of a Quadratic Function*
- Equation of the parabola in the intercept Form is $y = a(x-p)(x-q)$ where $(p, 0)$ and $(q, 0)$ are the *x*-intercepts. The *x*-coordinate of the vertex is $\frac{p+q}{2}$ and the equation of the axis of symmetry is

  $$x = \frac{p+q}{2}$$

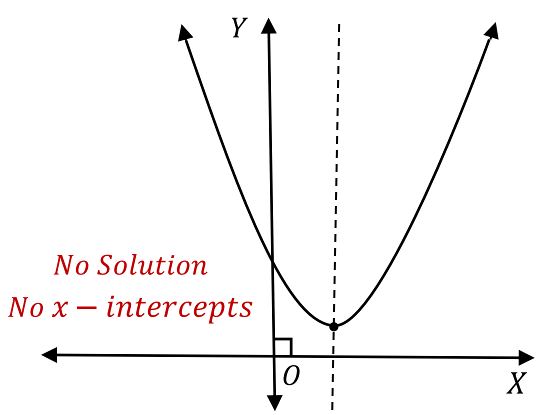
- The discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$ and the value of the discriminant determines the number and nature of the solutions
- If $b^2 - 4ac = 0$, there is one real solution. (Graphically speaking, the parabola touches the *X*-axis at the vertex and has only one $x -$ intercept which is the $x -$ coordinate of the vertex.



- If $b^2 - 4ac > 0$, there are two real solutions.
  (Graphically speaking, the parabola crosses the $X$-axis at two points and has two $x-$intercepts. )




- If $b^2 - 4ac < 0$, there is no real solution. It means that both solutions are complex conjugates of each other $(a \pm bi)$
  (Graphically speaking, the parabola does not touch or cross the $X$-axis and has no $x-$intercepts)



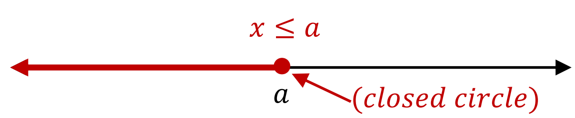

- If the value of $b^2 - 4ac$ is a perfect square, then the two solutions are rational and if it is not a perfect square then they are conjugate irrational numbers $\left(p \pm \sqrt{q}\right)$
- If $ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b$ and $c$ are real numbers, then the

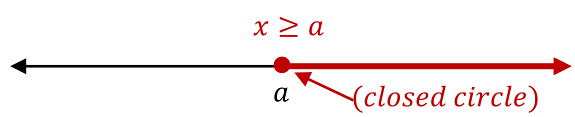
$$\text{Sum of the roots} = -\frac{b}{a}$$

$$\text{Product of the roots} = \frac{c}{a}$$

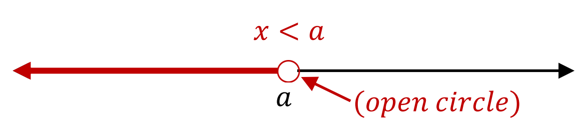
## ❖ INEQUALITIES

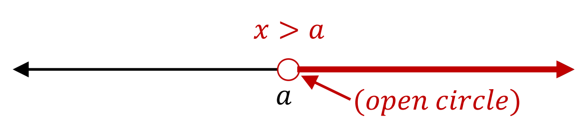
- Graphing one-variable inequalities –




- Closed circle (•) because of the = sign in the inequality $\leq \geq$




- Open circle (○) because there is no = sign in the inequality $< >$

- Converting LESS THAN Absolute-Value Inequality into a Sandwich /AND Compound Inequality

  If $|\,x\,| \le a$, then $-a \le x \le a$.

  Alternatively, $x \ge -a$ and $x \le a$


- Converting GREATER THAN Absolute-Value Inequality into a TWO-TAILS/OR Compound Inequality

  If $|\,x\,| \ge a$, then $x \le -a$ or $x \ge a$.


- If $|\,x\,| \ge -a$, then $x$ is any real number and there are infinitely many solutions since absolute value is always zero or positive and hence always greater than a negative number
- If $|\,x\,| \le -a$, then there is no solution since absolute value is always zero or positive and hence never less than or equal to a negative number
- Converting LESS THAN Compound Inequality into an Absolute-Value Inequality

  $|\,Variable - Midpoint\,| \le ½(Range)$

- Converting GREATER THAN Compound Inequality into an Absolute-Value Inequality

  $|\,Variable - Midpoint\,| \ge ½(Range)$

- Complement of $<$ is $\ge$
- Complement of $>$ is $\le$
- Complement of $\le$ is $>$
- Complement of $\ge$ is $<$

## ❖ GRAPHING LINEAR INEQUALITIES

- To graph the linear inequality, we first convert it in the slope-intercept form $y \le mx + b$ or $y \ge mx + b$. Then we graph the line $y = mx + b$. If the inequality is without the $=$ sign ($<$ or $>$) then draw a dashed (-----) line and if the inequality is with the $=$ sign ($\le$ or $\ge$) then draw a solid line (________).
- If the inequality is less than type, then shade the area of the coordinate plane that contains the lower part of the $Y$-axis.
- If the inequality is greater than type, then shade the area of the coordinate plane that contains the upper part of the $Y$-axis.

  TIP: If this question is asked in the Calculator section, then you may use the

graphing calculator for accuracy and saving time.

## ❖ VARIATION

- Direct Variation or Direct Proportion between two variables ($x$ and $y$) is when the two variables change in the same direction – both increase or both decrease.
  - If $x\uparrow$ then $y\uparrow$ and if $x\downarrow$ then $y\downarrow$
  - Equation of Direct Variation is $y = k \cdot x$
  - Constant of Variation is $k = \frac{y}{x}$ (Quotient is Constant)
  - Proportion is set by dividing the pair of variables $\frac{y_1}{x_1} = \frac{y_2}{x_2}$
- Inverse (Indirect) Variation or Inverse Proportion between two variables ($x$ and $y$) is when the two variables change in the opposite direction – one increases then the other decreases.
  - If $x\uparrow$ then $y\downarrow$ and if $x\downarrow$ then $y\uparrow$
  - Equation of Inverse Variation is $y = \frac{k}{x}$
  - Constant of Variation is $k = x \cdot y$ (Product is Constant)
  - Proportion is set by multiplying the pair of variables $x_1 \cdot y_1 = x_2 \cdot y_2$

## ❖ RATE – TIME – DISTANCE

- $Distance = Rate \cdot Time$
- $Rate = \frac{Distance}{Time}$
- $Time = \frac{Distance}{Rate}$
- $Average\ Speed = \frac{Total\ Distance}{Total\ Time}$

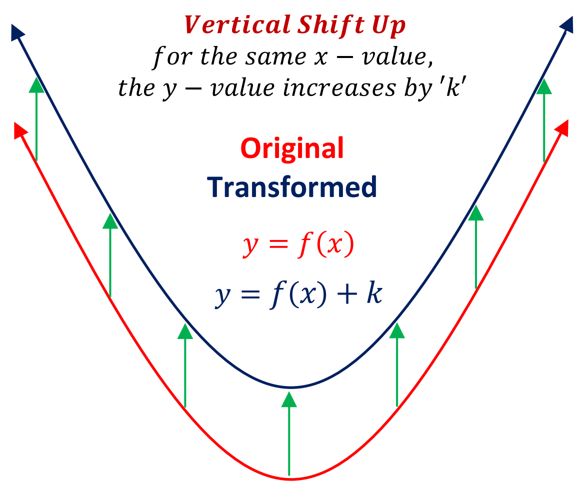
## ❖ TRANSFORMATIONS

**Original Graph is RED**

**Transformed Graph is BLUE**

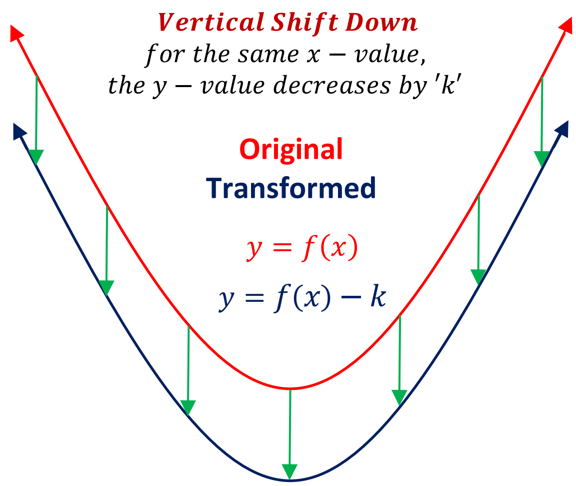
- If $y = f(x)$ is the parent function, then
  - Translations
    - Transformed Equation: $y = f(x) + k$

      *(Vertical shift up by 'k' units)*



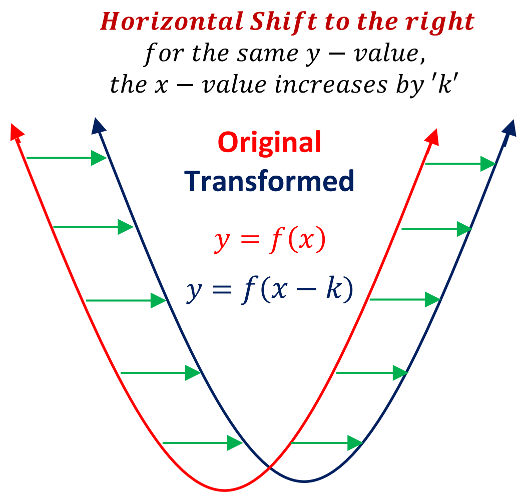
- Transformed Equation: $y = f(x) - k$

  *(Vertical shift down by 'k' units)*



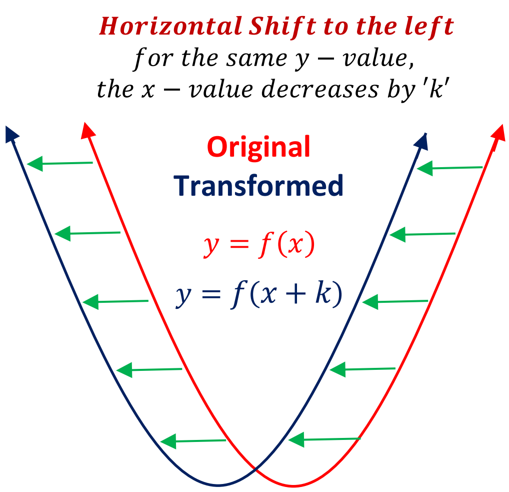

- Transformed Equation: $y = f(x - k)$

  *(Horizontal shift to the right by 'k' units)*



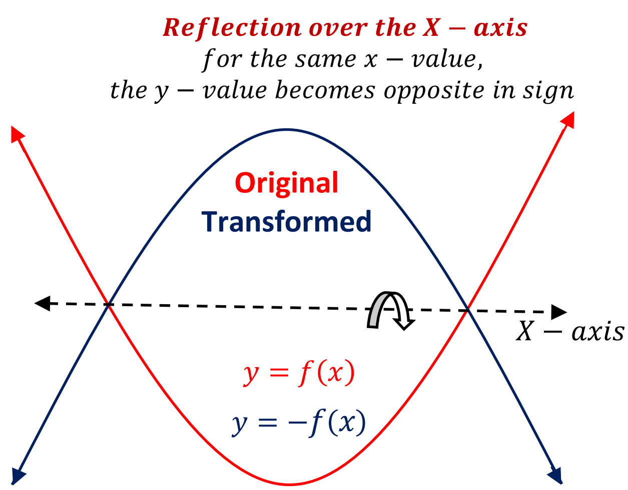

- Transformed Equation: $y = f(x + k)$

  *(Horizontal shift to the left by 'k' units)*



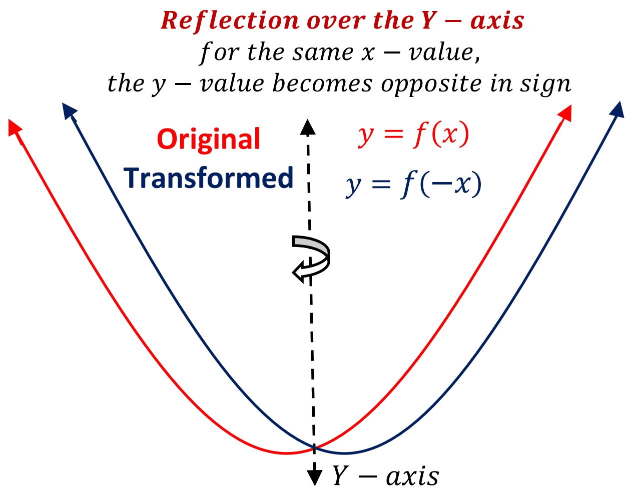

- Reflections
  - Transformed Equation: $y = -f(x)$

    *(Reflection over the X-axis)*



- Transformed Equation: $y = f(-x)$

*(Reflection over the Y-axis)*

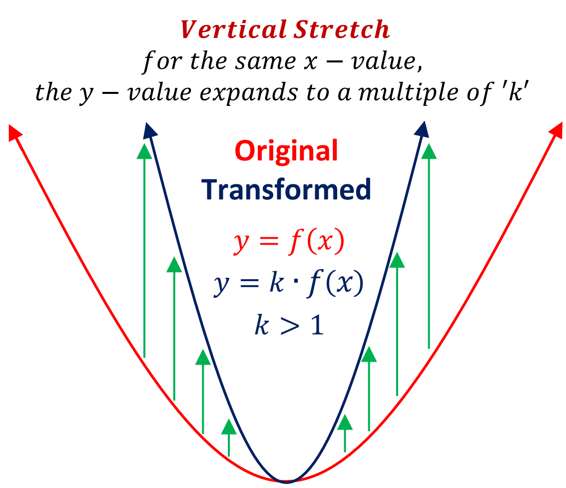



○ **DILATION**

- Transformed Equation: $y = k \cdot f(x)$

$(k > 1)$

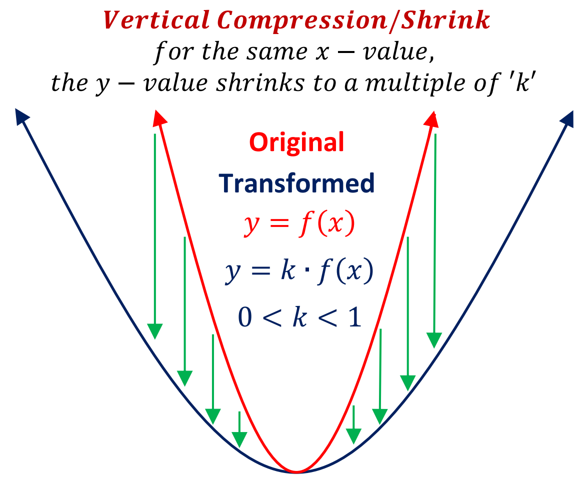
*(Vertical stretch by a factor of 'k')*




- Transformed Equation: $y = k \cdot f(x)$

$(0 < k < 1)$

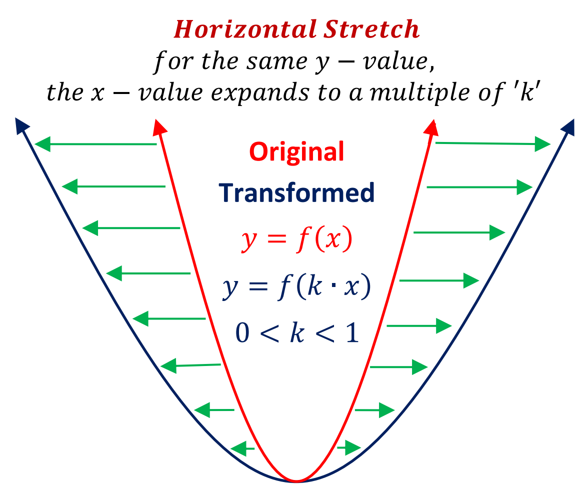
*(Vertical shrink by a factor of 'k')*




- Transformed Equation: $y = f(k \cdot x)$

$(0 < k < 1)$

*(Horizontal stretch by a factor of '$\frac{1}{k}$')*

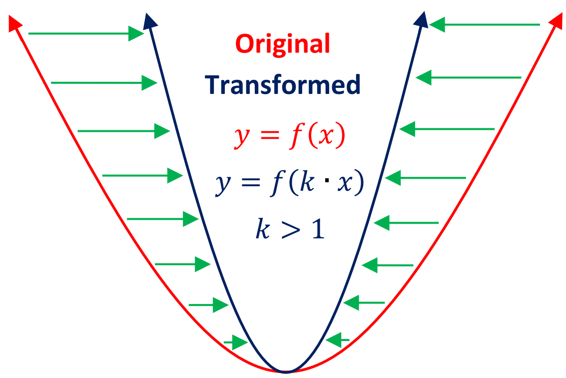


- Transformed Equation: $y = f(k \cdot x)$

  $(k > 1)$

  *(Horizontal shrink by a factor of '$\frac{1}{k}$')*

***Horizontal Compression/Shrink***
*for the same $y-$value,*
*the $x-$value shrinks to a multiple of 'k'*




## ❖ COMPLEX NUMBERS

- A number $a + ib$ is a complex number where $a$ and $b$ are real numbers and $i$ is the imaginary unit. ($i = \sqrt{-1}$). '$a$' is the real part of the complex number and '$ib$' is the imaginary part.
- Higher powers of $i$
  - $i^2 = -1$
  - $i^3 = i^2 \cdot i = -1 \cdot i = -i$
  - $i^4 = (i^2)^2 = (-1)^2 = 1$
  - $i^5 = i^4 \cdot i = 1 \cdot i = i$ … … … … and the cycle repeats after every four powers of $i$.
- **Adding Complex Numbers:** Two complex numbers $a + ib$ and $c + id$ can be added/subtracted by adding/subtracting their real parts and imaginary parts separately.

  $(a + ib) \pm (c + id)$

  $= (a \pm c) + i(b \pm d)$
- **Multiplying Complex Numbers:** Two complex numbers $a + ib$ and $c + id$ can be multiplied by FOIL

  $(a + ib) \cdot (c + id)$

  $= ac + a \cdot id + ib \cdot c + i^2 \cdot bd$

  $= ac + (ad + bc)i + (-1) \cdot bd$

  $= ac + (ad + bc)i - bd$

  $= (ac - bd) + (ad + bc)i$
- **Conjugate of a Complex Number:** The conjugate of a complex number $a + ib$ is $a - ib$ and vice versa. The real part remains the same and the sign of the imaginary part becomes opposite.
- **Dividing Complex Numbers:** A complex number $a + ib$ can be divided by another complex number $c + id$ by multiplying and dividing by the conjugate of $c + id$ which is $c - id$ and then do FOIL and simplification as shown in the following example:

$$\frac{5+2i}{3-4i} = \frac{5+2i}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{15+20i+6i+8i^2}{9-16i^2}$$

$$= \frac{15+20i+6i+8i^2}{9-16i^2} = \frac{15+26i+8(-1)}{9-16(-1)}$$

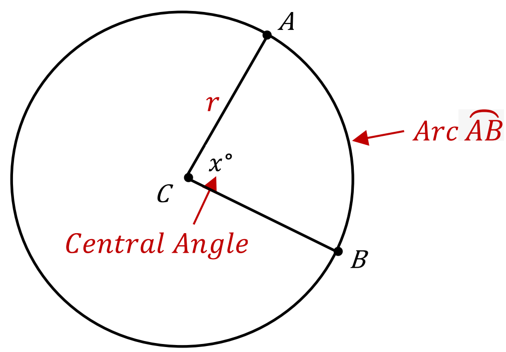
$$= \frac{15 + 26i - 8}{9 + 16} = \frac{7 + 26i}{25} = \frac{7}{25} + \frac{26i}{25}$$

## ❖ POLYNOMIALS

- If the graph of the polynomial function $y = f(x)$ passes through the point $(a, b)$ then the coordinates of the point $(a, b)$ satisfy the equation $y = f(x)$. We can plug the values of the coordinates in the equation of the polynomial function $y = f(x)$ and get $b = f(a)$. This strategy is useful to find values in many questions.
- "$c$ is a zero of the function $f$" means:
  - $x = c$ is the root or the solution of the equation $f(x) = 0$
  - $(x - c)$ is a factor of $f(x)$
  - The graph of $y = f(x)$ intersects the $x$-axis at $(c, 0)$
- REMAINDER THEOREM: If the polynomial function $f(x)$ is divided by the linear expression $(x - a)$ then the remainder is $f(a)$
- FACTOR THEOREM: If the polynomial function $f(x)$ is divided by the linear expression $(x - a)$ and the remainder is zero, then $(x - a)$ is a factor of $f(x)$. So if $f(a) = 0$ means that $(x - a)$ is a factor of the polynomial function $f(x)$.

## ❖ CIRCLES

- **Length of the Arc of a Circle**



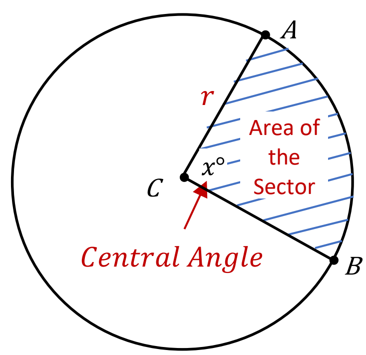

Length of the Arc

$$= \frac{m\angle(Central\ Angle)°}{360°} \times \text{Circumference}$$

$$= \frac{m\angle(Central\ Angle)°}{360°} \times 2\pi r$$

$$= \frac{x°}{360°} \times 2\pi r$$

- **Area of the Sector of a Circle**




- Area of the Sector

$$= \frac{m\angle(Central\ Angle)°}{360°} \times \text{Area of the Circle}$$

$$= \frac{m\angle(Central\ Angle)^\circ}{360^\circ} \times \pi r^2$$

$$= \frac{x^\circ}{360^\circ} \times \pi r^2$$

- **Using Proportions**

  In the form of a Proportion, we may express the above as follows:

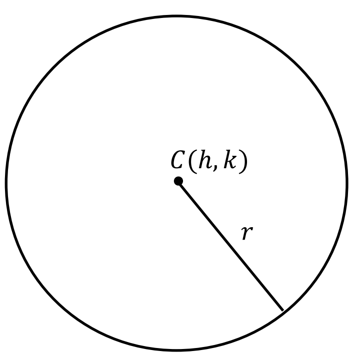
$$\frac{\text{Length of the Arc}}{\text{Circumference}} = \frac{m\angle(Central\ Angle)^\circ}{360^\circ}$$

$$\frac{\text{Area of the Sector}}{\text{Area of the Circle}} = \frac{m\angle(Central\ Angle)^\circ}{360^\circ}$$

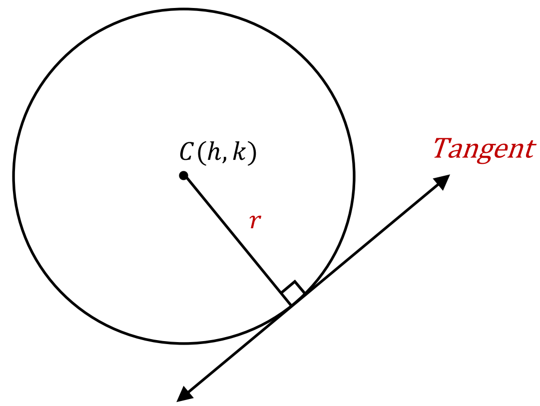
- If the measure of the central angle is given in terms of $\pi$, then the measure is usually in radians.
- **Equation of Circle in CENTER – RADIUS FORM:**

  Equation of Circle $\odot P$ with center as $P(h, k)$ and radius $= r$ is

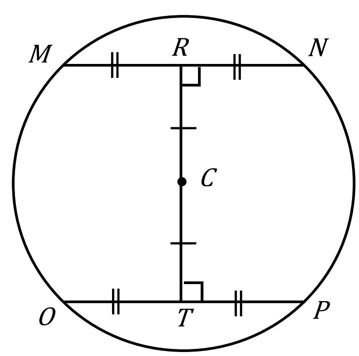
$$(x-h)^2 + (y-k)^2 = r^2$$



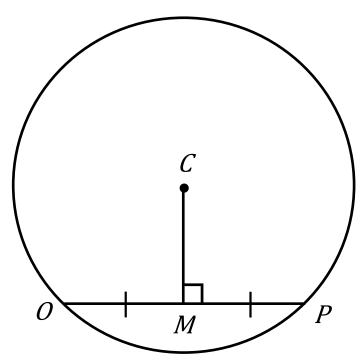

- **Important Rules:**
- A tangent to the circle is always perpendicular to the radius drawn from the center to the point of contact or the point of tangency



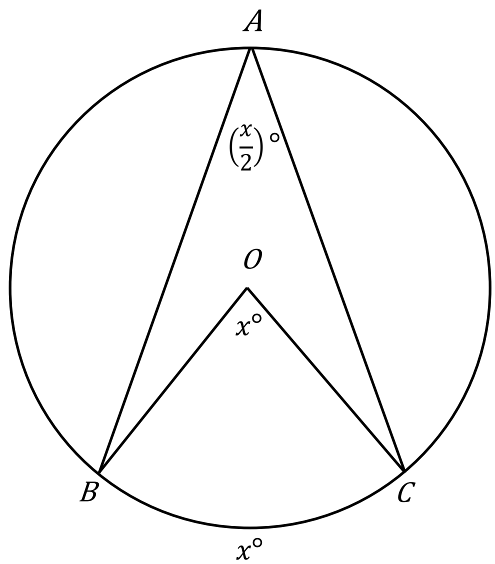

- Congruent chords are equidistant from the center of the circle




- Perpendicular from the center to the chord bisects the chord



- **Relationship between the Central Angle, Inscribed Angle, and Intercepted Arc**




Measure of the Central Angle $\angle BOC$

$=$ Measure of the Intercepted Arc $\widehat{AB}$

$= x^\circ$

Measure of the Inscribed Angle $\angle BAC$

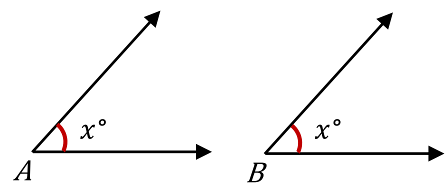
$=$ ½(Measure of the Intercepted Arc $\widehat{AB}$)

$= ½(x^\circ) = \left(\frac{x}{2}\right)^\circ$

## ❖ ANGLES

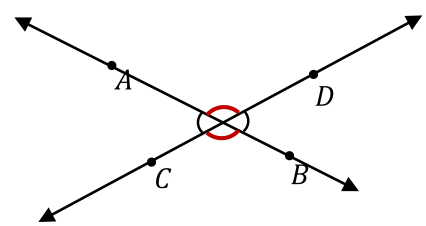
- If two angles are congruent, then their measures are equal. If $\angle A \cong \angle B$; then $m\angle A = m\angle B$

$\angle A \cong \angle B$
$m\angle A = m\angle B$



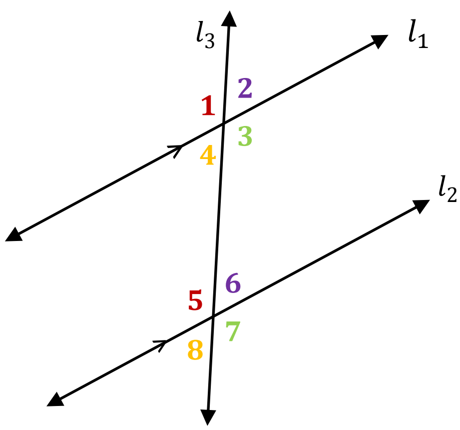

- Vertical Angles, also called opposite angles, are congruent

*Vertical Angles are conguent*




- If two parallel lines are cut by a transversal, then the following pairs of angles are congruent:
  - 4 pairs of Corresponding Angles are congruent
  - 2 pairs of Alternate Interior Angles are congruent
  - 2 pairs of Alternate Exterior Angles are congruent
- If two angles form a linear pair, the angles are supplementary, whose measures add up to 180°.

- If two parallel lines are cut by a transversal, then the following pairs of angles are supplementary:
  - 2 pairs of Same-Side or Consecutive Interior Angles are supplementary
  - 2 pairs of Same-Side or Consecutive Exterior Angles are supplementary




- The 4 pairs of congruent Corresponding Angles are:

$$\angle 1 \cong \angle 5;$$
$$\angle 2 \cong \angle 6;$$
$$\angle 3 \cong \angle 7;$$
$$\angle 4 \cong \angle 8;$$

- The 2 pairs of congruent Alternate Interior Angles are:

$$\angle 3 \cong \angle 5;$$
$$\angle 4 \cong \angle 6;$$

- The 2 pairs of congruent Alternate Exterior Angles are:

$$\angle 2 \cong \angle 8;$$
$$\angle 1 \cong \angle 7;$$

- The 2 pairs of supplementary Same-Side or Consecutive Interior Angles are:

$$m\angle 3 + m\angle 6 = 180°$$
$$m\angle 4 + m\angle 5 = 180°$$

- The 2 pairs of supplementary Same-Side or Consecutive Exterior Angles are:

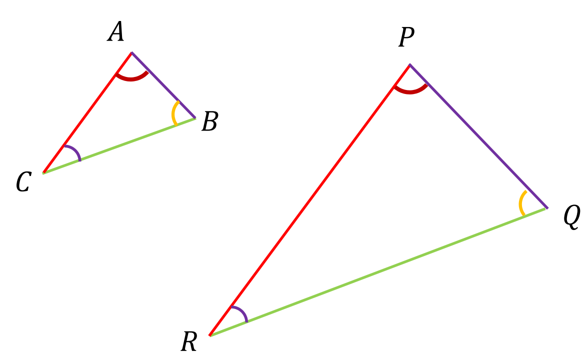
$$m\angle 1 + m\angle 8 = 180°$$
$$m\angle 2 + m\angle 7 = 180°$$

## ❖ TRIANGLES

In similar triangles, the corresponding angles are congruent, and the lengths of the corresponding sides are proportional. If $\triangle ABC \sim \triangle PQR$, then

$$A \cong \angle P;\ \angle B \cong \angle Q;\ \angle C \cong \angle R$$

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$



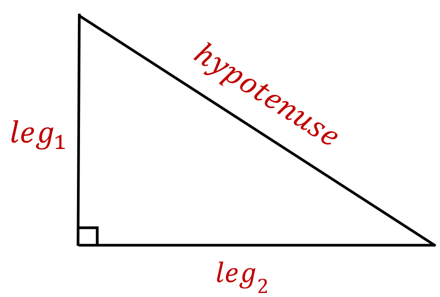
PYTHAGOREAN THEOREM: In any right triangle, the square of the length of the hypotenuse is always equal to the sum of the squares of the lengths of the legs.

$$(hypotenuse)^2 = (leg_1)^2 + (leg_2)^2$$

$$hypotenuse = \sqrt{(leg_1)^2 + (leg_2)^2}$$

$$leg_1 = \sqrt{(hypotenuse)^2 - (leg_2)^2}$$

$$leg_2 = \sqrt{(hypotenuse)^2 - (leg_1)^2}$$




- Famous Pythagorean Triples:

  $3 - 4 - 5; 6 - 8 - 10; 5 - 12 - 13$

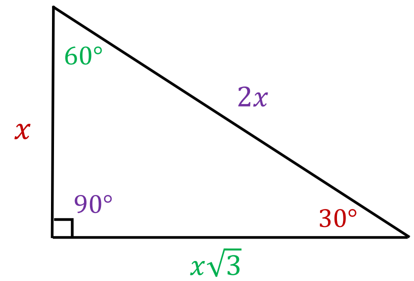
- Not-so-Famous Pythagorean Triples:

  $8 - 15 - 17; 7 - 24 - 25;\ 9 - 40 - 41$

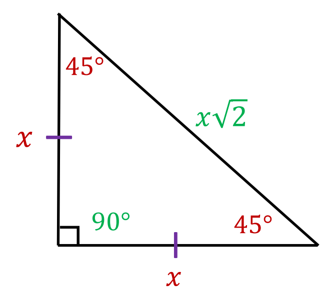
- In special right triangles:

  $30° - 60° - 90°$ and $45° - 45° - 90°$, there is a special relationship among the three sides as follows –

  - $30° - 60° - 90°$: If the side opposite to $30°$ is $x$, then the side opposite to $60°$ will be $x\sqrt{3}$ and the side opposite to $90°$ will be $2x$.




  - $45° - 45° - 90°$: If the side opposite to $45°$ is $x$, then the side opposite to $90°$ will be $x\sqrt{2}$.




❖ **GEOMETRY FORMULAS FOR PERIMETER, AREA, SURFACE AREA & VOLUME**

*Key to decode the Perimeter, Area, Surface Area, and Volume formulas –*

$s = side\ or\ edge$;

$l = length; w = width; h = height$;

$r = radius$;

$d = diameter\ or\ diagonal;$

$l = slant\ height$

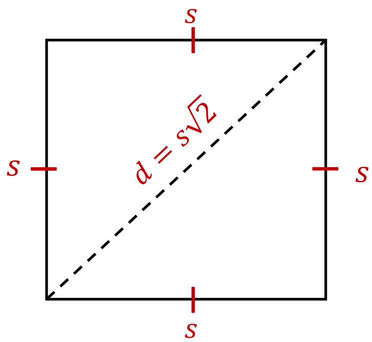
$C = Circumference\ of\ the\ Base$;

$P = Perimeter\ of\ the\ Base$;

$B = Area\ of\ the\ Base$

## 2 – DIMENSIONAL FIGURES

- **Square**

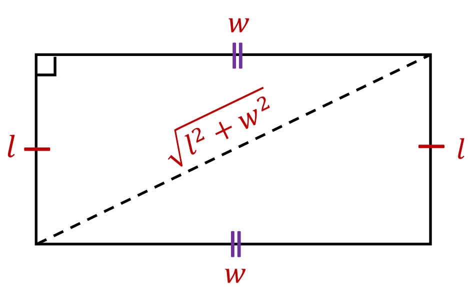



Perimeter of a Square $= 4s$

Diagonal of a Square $= d = s\sqrt{2}$

Area of Square $= s^2 = \frac{1}{2}d^2$

- **Rectangle**

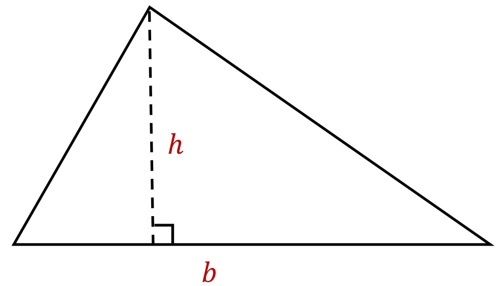



Perimeter of a Rectangle $= 2l + 2w$

Diagonal of a Rectangle $= d = \sqrt{l^2 + w^2}$

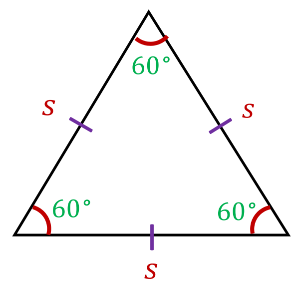
Area of a Rectangle $= l \cdot w$

- **Triangle**



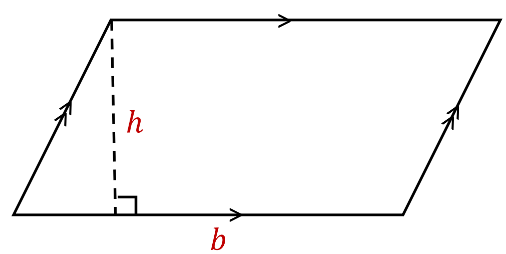

Area of a Triangle $= \frac{1}{2} \cdot b \cdot h$

- **Equilateral Triangle**



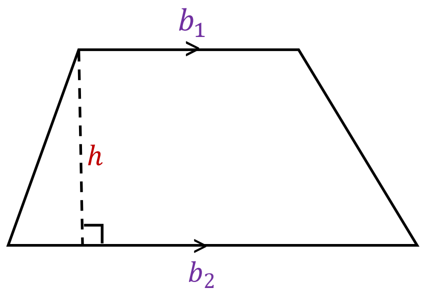

Area of an Equilateral Triangle $= \frac{\sqrt{3}}{4} \cdot s^2$

- **Parallelogram**



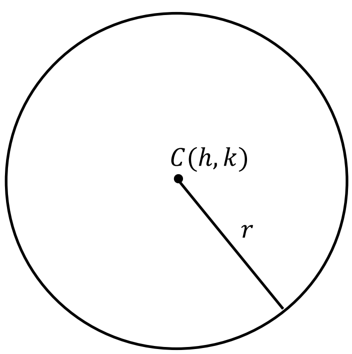

Area of a Parallelogram $= b \cdot h$

- **Trapezoid**




Area of a Trapezoid $= \frac{1}{2}(b_1 + b_2) \cdot h$

- **Circle**

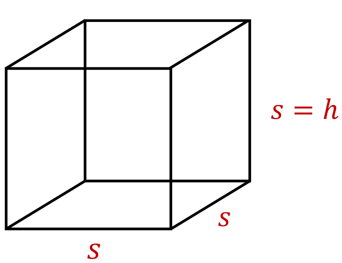



Circumference of a Circle $= 2\pi r = \pi d$

Area of a Circle $= \pi r^2$

## 3 – DIMENSIONAL FIGURES

- **Cube**



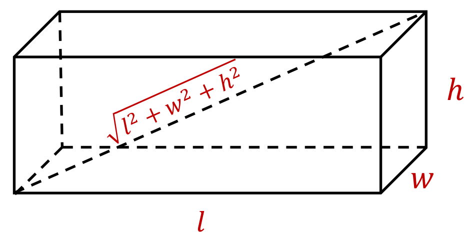

Lateral Area of a Cube $= 4s^2 = P \cdot h$

Surface Area of a Cube $= 6s^2 = 2B + P \cdot h$

Volume of a Cube $= s^3 = B \cdot h$

Length of a Diagonal of a Cube $= s\sqrt{3}$

- **Rectangular Prism**




Lateral Area of a Rectangular Prism $= P \cdot h$

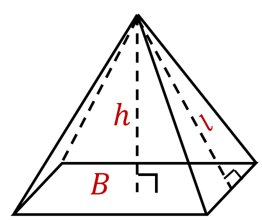
Surface Area of a Rectangular Prism

$= 2B + P \cdot h$

$= 2lw + 2lh + 2w$

Volume of a Rectangular Prism $= l \cdot w \cdot h$

- **Pyramid**



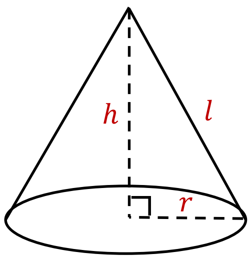

Lateral Area of a Pyramid $=$ ½ $\cdot P \cdot l$

Surface Area of a Pyramid

$= B +$ ½ $\cdot P \cdot h$

Volume of a Pyramid $= \frac{1}{3} \cdot B \cdot h$

- **Cone**

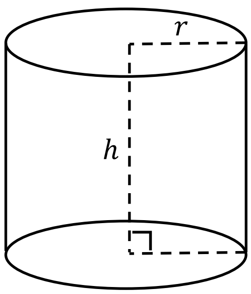



Lateral Area of a Cone $= ½ \cdot C \cdot h = \pi rl$

Surface Area of a Cone $= \pi r^2 + \pi rl$

Volume of a Cone $= \dfrac{1}{3}\pi r^2 h$

- **Cylinder**




Lateral Area of a Cylinder $= C \cdot h = 2\pi rh$

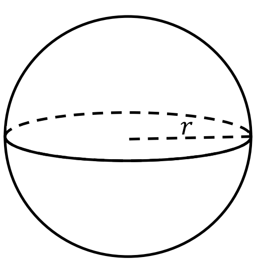
Surface Area of a Cylinder $= 2B + C \cdot h$

Surface Area of a Cylinder

$= 2\pi r^2 + 2\pi rh$

Volume of a Cylinder $= \dfrac{1}{3} \cdot B \cdot h = \pi r^2 h$

- **Sphere**




Lateral Area of a Sphere $= ?$

$(No\ such\ concept!\ Think\ why?)$

Surface Area of a Sphere $= 4\pi r^2$

Volume of a Sphere $= \dfrac{4}{3}\pi r^3$

❖ **Ratios of Perimeter, Area & Volume**

If the length of the sides of two geometrical figures is in the ratio of $a : b$ then the

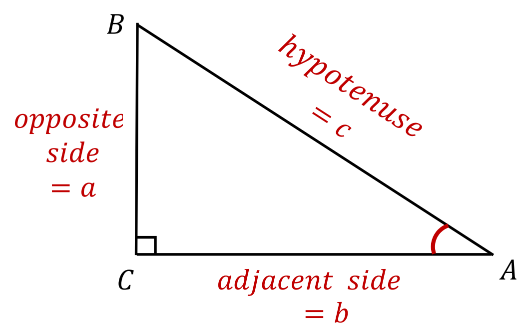
Ratio of Perimeters is $a : b$

Ratio of Areas is $a^2 : b^2$

Ratio of Volume is $a^3 : b^3$

❖ **TRIGONOMETRY**

In a *right triangle* $\triangle ABC$, right angled at $C$, means $m\angle C = 90°$




$$\sin A = \frac{opposite\ side}{hypotenuse} = \frac{a}{c}$$

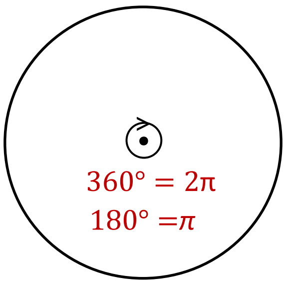
$$\cos A = \frac{adjacent\ side}{hypotenuse} = \frac{b}{c}$$

$$\tan A = \frac{opposite\ side}{adjacent\ side} = \frac{a}{b}$$

## ❖ CONVERTING DEGREES INTO RADIANS

To convert $(x°)$ degrees to radians, multiply $x°$ by $\frac{\pi}{180}$

$$x° = x \cdot \frac{\pi}{180} radians$$




## ❖ CONVERTING DEGREES INTO RADIANS

To convert $(x)$ radians to degrees, multiply $x$ by $\frac{180}{\pi}$

$$x \, radians = \frac{180}{\pi} \cdot x°$$

## ❖ DEGREES AND RADIAN RELATIONSHIP OF SOME SPECIAL ANGLES

Degrees and Radian Relationship of some Special Angles:

- $180° = \pi \, radians$
- $270° = \frac{3\pi}{2} \, radians$
- $360° = 2\pi \, radians$
- $30° = \frac{\pi}{6} radians$
- $45° = \frac{\pi}{4} radians$
- $60° = \frac{\pi}{3} radians$
- $90° = \frac{\pi}{2} radians$

## ❖ Cofunction Relationship

- $\sin x° = \cos(90 - x)°$
- $\cos x° = \sin(90 - x)°$
- If $\sin A° = \cos B°$, then $A + B = 90$
- In radians, $\sin x = \cos\left(\frac{\pi}{2} - x\right)$
- In radians, $\cos x = \sin\left(\frac{\pi}{2} - x\right)$
- In radians, $\sin A = \cos B$,

  then $A + B = \frac{\pi}{2}$
- In words, Sine of an angle is equal to the Cosine of the complementary angle and vice-versa.
- The cofunction relationship comes from the fact that the side opposite to angle $A$ is the same as the side adjacent to angle $B$ and vice-versa.